LATTICE UNIT-3

Partial order set (POSET) : →

A non-empty set A, together with a binary relation R is said to be a partial order set or Poset if the following conditions are satisfied –

(i) Reflexive :– $(a,a) \in R \quad \forall\, a \in A$
i.e $aRa \quad \forall\, a \in A$.

(ii) Antisymmetric :– If $a, b \in A$ Then
$(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$
i.e aRb & $bRa \Rightarrow a = b$.

(iii) Transitive :– $a, b, c \in A$
$(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$
i.e aRb & $bRc \Rightarrow aRc$.

→ The Relation R on Set A is called Partial order Relation.
Then Poset is ~~defined~~ denoted by (A, R).

EX-1 Let $A = \{2, 3, 6, 12, 24, 36\}$ and R be the Relation in A is defined by a divides b or a/b. Then prove that R is a Partial order Relation.

Soln:– Let $aRb = a/b \quad \left(\frac{b}{a}\right)$

(i) Reflexive :→ $\because a/a \Rightarrow aRa \quad \forall\, a \in A$.
$\Rightarrow R$ is Reflexive.

(ii) Antisymmetric :→ If a/b and $b/a \Rightarrow a = b$
$\therefore aRb$ and $bRa \Rightarrow \boxed{a = b}$
$\Rightarrow R$ is Antisymmetric.

(iii) Transitive :– Let $a, b, c \in A$
and a/b and $b/c \Rightarrow a/c$
$\Rightarrow aRb$ & $bRc \Rightarrow aRc \Rightarrow R$ is Transitive.

Hence R is the Partial order Relation.
& therefore (A, R) is a Partial order Relation Set or Poset. P.T.O.

Representation of Posets or (Hasse Diagram.)

a graphical representation of a Partial order Relation in which all arrow heads are understood to be Pointing upward is known as the Hasse Diagram.

→ **Procedure for Drawing Hasse diagram :→**

Step -1 Draw Digraph of the Relation. (Directed Graph)

Step-2 Remove self-loops.

Step-3 Remove all Transitive edges

Step-4 Arrange all edges Pointing upwards and Remove arrows from edges.

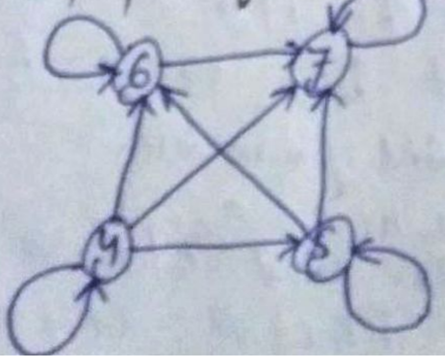
Step-5 Replace circles by dots or vertex.

Ex:-1 consider the set $A = \{4, 5, 6, 7\}$. Let R be the relation $\leq$ on A. Draw the Hasse Diagram of R.

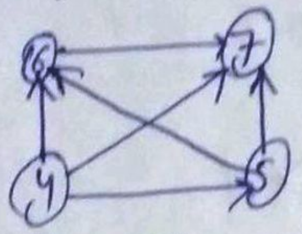
Solⁿ:- Given $A = \{4, 5, 6, 7\}$ and $R \rightarrow \leq$.

Then $R = \{(4,4), (5,5), (6,6), (7,7), (4,5), (4,6), (4,7), (5,6), (5,7), (6,7)\}$.

Step-1 Directed Graph of R →



Step 2 – Remove self-loops,



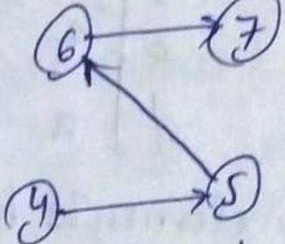
Step 3 – Remove Transitive edges:

$\{(4,5), (4,6), (4,7), (5,6), (5,7), (6,7)\}$

$(4,5) \in R$ & $(5,6) \in R \Rightarrow (4,6) \in R$

$(4,5) \in R$ & $(5,7) \in R \Rightarrow (4,7) \in R$

$(5,6) \in R$ & $(6,7) \in R \Rightarrow (5,7) \in R$.



Step 4 → Remove arrow by edges and Replace circle by dot

It is Required Hasse Diagram.

LATTICE :– A Partial order set (Poset) (L, R) is a lattice if $\forall\ a, b \in L$ $\sup\{a,b\}$ and $\inf\{a,b\}$ exists in L.

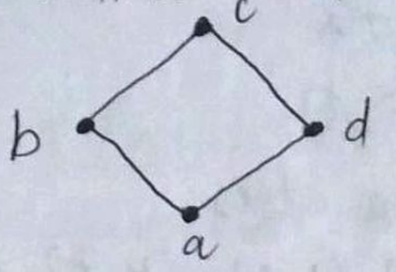
Note :– ① $a \vee b$ = a join b = $\sup\{a,b\}$ = $lub\{a,b\}$

② $a \vee b$ = a meet b = $\inf\{a,b\}$ = $glb\{a,b\}$.

where lub = least upper bound = supremum

& glb = greatest lower bound = ~~glb~~. Infimum

Ex:-1. Determine whether the following Hasse diagram represent lattice or not.



Solⁿ:-

① lub Table →

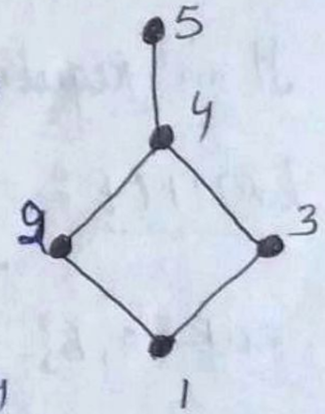
∨	a	b	c	d
a	a	b	c	d
b	b	b	c	c
c	c	c	c	c
d	d	c	c	d

② glb Table →

∧	a	b	c	d
a	a	a	a	a
b	a	b	b	a
c	a	b	c	d
d	a	a	d	d

Since each subset of two elements has least upper bound and a greatest lower bound. So this is a lattice

Ex-2. Determine whether the following Hasse Diagram represent lattice or not.



Solⁿ:-

lub Table →

∨	1	2	3	4	5
1	1	2	3	4	5
2	2	2	4	4	5
3	3	4	3	4	5
4	4	4	4	4	5
5	5	5	5	5	5

glb Table →

∧	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	2
3	1	1	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Since each subset of two elements has least upper bound and greatest lower bound.

So this is a lattice

Greatest and least element of a POSET :→

* Let (P, R) be a Poset. Then an element $a \in P$ is the greatest element of P if xRa, $\forall x \in P$.
  i.e every element in P precedes a.

→ The Greatest element, if it exist, is unique.

* an element $b \in P$ is called the least element of P if bRx, $\forall x \in P$. i.e every element in P succeeds b.

→ The least element, if it exist, is unique.

Maximal and minimal element of a Poset :→

an element a in the Poset (P, R) is called a maximal element of P if aRx, for no x in P.
i.e if no element of P strictly succeeds a.

Similarly, an element $b \in P$ is called a minimal element of P if xRb, for no x in P.

Note:- 1. Maximal and minimal elements are easy to spot using a Hasse Diagram. They are the top and bottom elements in the Diagram.

2. The minimal and maximal element of a Poset need not be unique.

Ex:- $P = \{2, 3, 4, 6, 12\}$. (divisors of 12).
$R \rightarrow$ divisibility

here 2 and 3 are minimal elements
& 2 & 3 are not comparable.

$P = \{2, 3, 4, 6, 12, 24, 36\}$.
R = divisibility.
here 24 & 36 are maximal elements & they are not comparable.

(3) It is not necessary that Every Poset has a maximal and minimal element.

Ex:- The natural no.s under usual $\leq$ has no maximal element.

upper and lower bounds of a Poset :→

Let (P, R) be a Poset, & let A be any subset of P. Then an element a of P is said to be the upper bound of A if xRa, $\forall x \in A$.

an element b of P is said to be lower bound of A if bRx, $\forall x \in A$.

Supremum & Infimum :→

The least element of the set of upper bound is said to be least upper bound (supremum) of A.

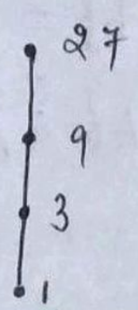
and the Greatest element of the set of lower bound is said to be greatest lower bound (Infimum). of A.

Note:- (1) The greatest element is always the supremum but the converse is not true. In fact, $a = \sup(A)$ is the greatest element iff $a \in B$.

(2) The least element is always the ~~supremum~~ Infimum but the converse is not true.
In fact $b = \inf(A)$ is least element iff $b \in A$.

Ex:- Determine whether the Posets Represents by Hasse Diagram have a greatest element, least element, minimal element and maximal elements.

(a)



R → |

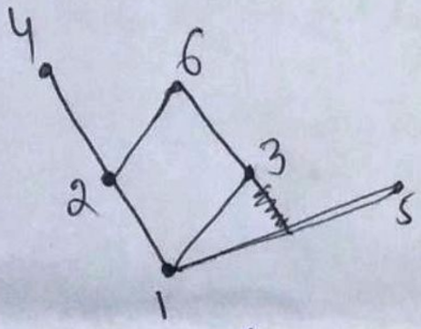
∵ 1|27, 3|27, 3|27 & 9|27

⇒ 27 is the greatest element of and is the only maximal element.

∵ 1|1, 1|3, 1|9, 1|27

⇒ 1 is the least element of Poset & is the only ~~Maxi~~ minimal element.

(b)



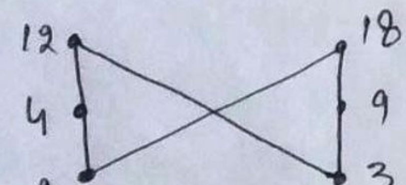
R = ≤

There is no greatest element.

4, 6 & 5 are maximal elements

least element of Poset is 1 & is the only minimal element.

(c)



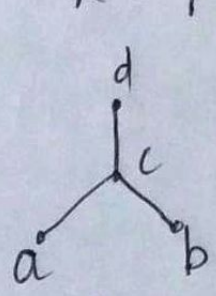
R = |

The Poset with this Hasse Diagram has neither greatest element nor least element.

2 and 3 are minimal elements

12 & 18 are maximal elements

(d)

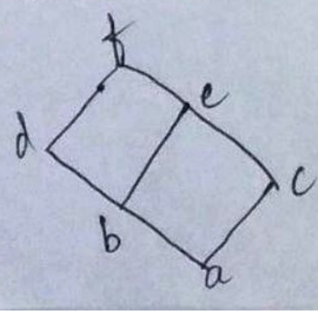


greatest & maximal element = d

minimal elements = a, b

least element - no

(e)



greatest element = f = Maximal ele.

least element = a = minimal element

Ex:- In the Poset $A = (\{1, 2, 3, 4, \cdots, 10\}, |)$
the Subset $B = \{2, 7\}$ has no upper bound
since there is no Integer in A which is divisible
by both 2 and 7. The lower of B is 1 since
$1|2$ & $1|7$. Hence 1 is the greatest lower bound
for $\{2, 7\}$ i.e $glb\{2, 7\} = 1$.
The Subset $\{1, 2, 3\}$ has 6 and 1 as unique upper
and lower bounds. Hence $lub\{1, 2, 3\} = 6$ &
$glb\{1, 2, 3\} = 1$.
The subset $\{1, 2, 4\}$ has 4 and 8 as upper bounds.
Hence $lub\{1, 2, 4\} = 4$.

Ques. Find the maximal and minimal elements of the Poset $(P, \leq)$, where $P = \{2, 4, 5, 10, 12, 20, 25\}$ and the Partial order $\leq$ is given by the relation 'divides'.

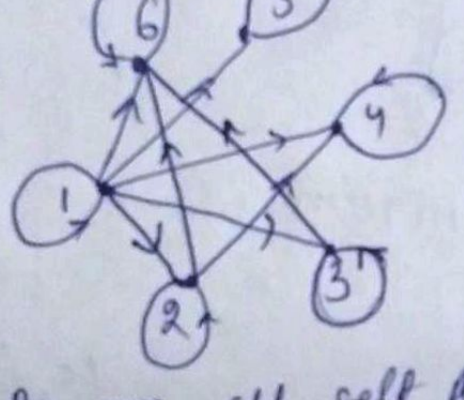
Ques. Let $P = (\{1, 2, 3, 4, 5, 6, 7, 8\}, |)$ be a Poset
find the lower and upper bounds of the subsets $A = \{1, 2\}$ and $B = \{3, 4, 5\}$ of $(P, \leq)$
Also find the $Sup(A)$, $inf(A)$, $sup(B)$, and $inf(B)$

Ques. Let $P = \{a, b, c\}$. Draw the Hasse diagrams of all possible Poset structures that P may have. Also, Discuss their maximal, minimal elements.

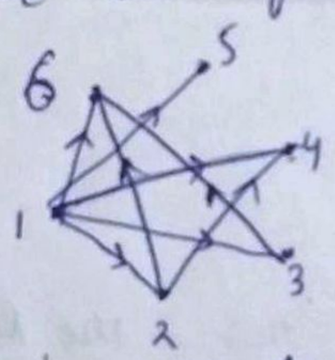
Ques. Draw the Hasse Diagram of Poset $(X, |)$ where $X = \{1, 2, 3, 4, 5, 6\}$.

Sol^n:- Step I:- Draw the Digraph of R

$$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)\}.$$

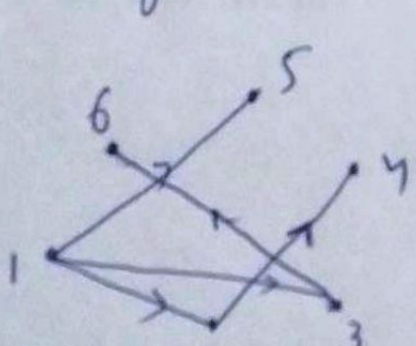


Step 2: Remove all self loops.

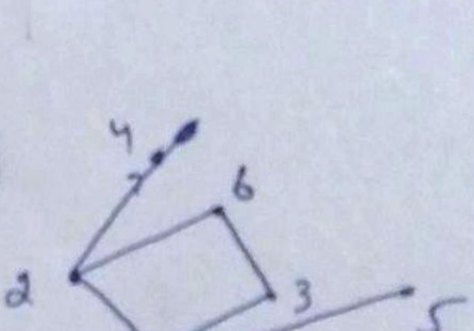


(1, 4)
(1, 6)

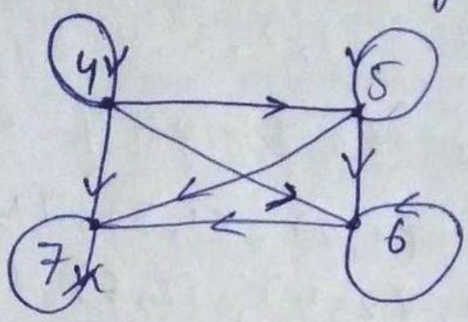
Step 3:- Removing all transitive edges



(4)

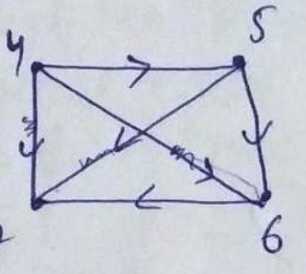


Step 1   Digraph (Directed graph) of R



$$\left[\begin{array}{l} A = \{4, 5, 6, 7\} \\ R \to \le \text{ on } A \end{array}\right]$$

Step 2 : –   Removing all self loops.

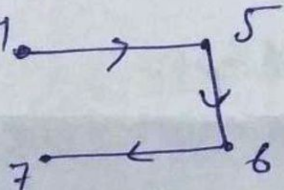


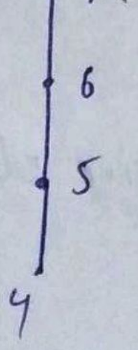
$\{(4,5), (4,6), (4,7),$
$(5,6), (5,7), (6,7),\}.$

$(4,5)$ & $(5,6)$
$\Rightarrow (4,6)$

Step 3: –   Remove all Transitive edges.

$(4,5)$ & $(5,7)$
$\Rightarrow (4,7)$
$(5,6)$ & $(6,7)$
$\Rightarrow (5,7)$.

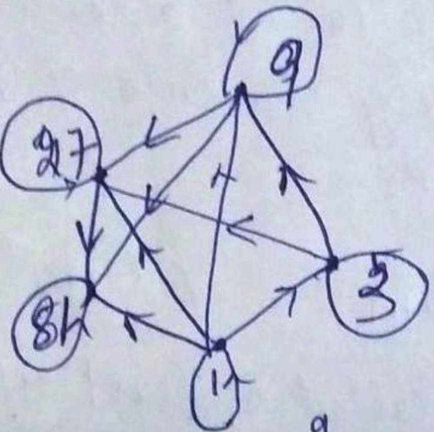




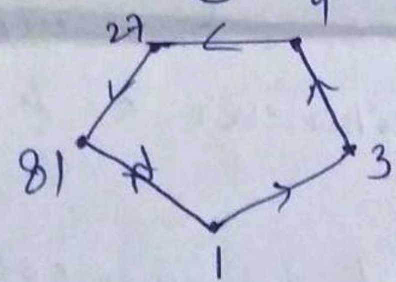
This is the Required Hasse Diagram of R.

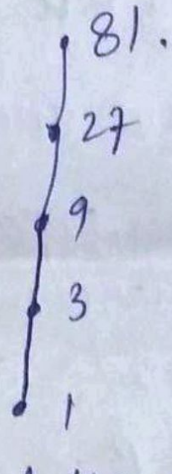
Qn. Let $A = \{1, 3, 9, 27, 81\}$.
draw the Hasse Diagram of the Poset $(A, |)$

Soln.:- ① $R = \{(1,1), (1,3), (1,9), (1,27), (1,81), (3,3), (3,9), (3,27), (3,81), (9,9), (9,27), (9,81), (27,27), (27,81), (81,81)\}$.



② & ③





Required Hasse Diagram of $(A, |)$.

Karnaugh Map (K-map): →

→ K-Map is developed by Maurice Karnaugh (1924) in 1953.

→ K-map is used to simplify the boolean expression in ~~Pictorial Format~~ without using boolean law or theorems.

→ The number of cells Required in K-map $= 2^n$.
where n is the no. of variables in a Boolean function.

→ K-map is based on Grey code i.e unit distance code.

→ K-map is based on three types of Inputs values (0, 1, don't care).

→ K-map is useful if the Boolean Expression contains 6 or less variables.

* Rules For K-map Simplification: →

→ zeros are not allowed in creating groups.

→ Group can be vertical or Horizontal but can't be diagonal.

→ overlapping of Groups is allowed.

→ Group should be as large as possible.

→ Group must contain 2^n cells. i.e 1, 2, 4, 8 ...

Case I - For two variables: $n = 2$, $2^2 = 4$ cells presents

	y	y'
x	xy	xy'
x'	$x'y$	$x'y'$

A \ B		
	0	1
	3	2

A \ B	0	1
0		
1		

$A = 1$

$A' = 0$.

Ex:- find the simplify form of the Boolean Expression using K-map.

$$F(x, y) = xy + xy' + x'y'$$

Sol^n:- $2^2 = 4$ cells

	y	y'
x	xy	xy'
x'	$x'y$	$x'y'$

	y	y'
x	1	1
x'	0	1

$F(x, y) = x + y'$

Ex-2. $F(x, y) = \Sigma(0, 2, 3)$

The cells Required in K-map $= 2^2 = 4$.

x \ y		
	0	1
	3	2

x \ y	0	1
0	1	0
1	1	1

$x' = 0 = y'$

$x = 1 = y$

$$F(x, y) = x + y'$$

Case II - For three Variables : $n = 3$, no. of cells $= 2^2 = 8$.

x \ yz	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Ex-1 $F(x, y, z) = \Sigma(0, 1, 2, 3, 5)$.

no. of cells $= 2^3 = 8$.

x \ yz	00	01	11	10
0	1	1	1	1
1	0	1	0	0

$x' = y' = 0$

$x = y = 1$

x \ yz				
	0	1	3	2
	4	5	7	6

$$F(x, y, z) = x' + y'z$$

Ex-2. $F(x, y, z) = \Sigma(0, 2, 3, 7)$.

x \ yz	00	01	11	10
0	1	1	1	
1			1	

$$F(x, y) = x' + yz$$

Case-3. for 4-variables :-

no. of cells Required $= 2^4 = 16$.

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD				
	0	1	3	2
	4	5	7	6
	12	13	15	14
	8	9	11	10

Ex:-1 $f(x, y, z, w) =$

$f(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

wx \ yz	00	01	11	10
00	1	1	0	1
01	1	1		1
11	1	1		1
10	1	1		

III

II

I

$F(x, y, z) = y' + xz' + w'z'$

Ex:-2 ~~Find~~ Simplify the Boolean function

$F(a, b, c, d) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$.

ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	1	0

I

II II

$a = 1$
$b = 1$
$c = 1$
$d = 1$

$a' = b' = c' = d' = 0$.

ab \ cd	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$F(a, b, c, d) = a' + b'c' + b'd$.

Ex: Find the minimal SOP Expression for the function

$F(u,v,w) = uv'w' + uvw' + uvw + u'v'w.$

The No. of cells Required in K-map $= 2^3 = 8$.

u \ vw	vw	vw'	v'w'	v'w
u	uvw	uvw'	uv'w'	uv'w
u'	u'vw	u'vw'	u'v'w'	u'v'w

u \ vw	vw	vw'	v'w'	v'w
u	1	1	1	
u'				1

$F(u,v,w) = uv + uv'w' + u'v'w.$

Ex: $F(x,y,z,w) = \Sigma(1,3,5,7,9,11,13,15)$

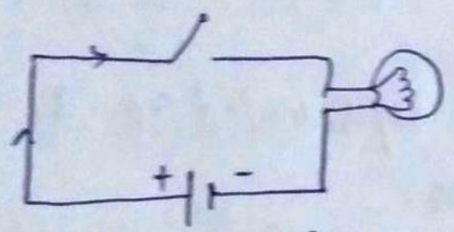
xy \ zw	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10		1	1	

$w = 1$

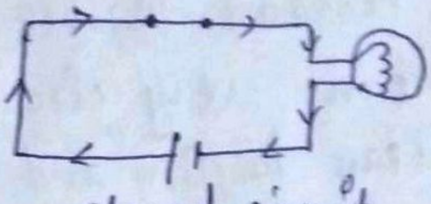
$F(x,y,z,w) = w.$

7) Applications of Boolean Algebra :→

Switches:→ A switch in a circuit is a device which is either in close form or open form (on or off Respectively) If Switch is in close form, then current flows by the switch. If switch is in open form, then current does not flow by the switch.



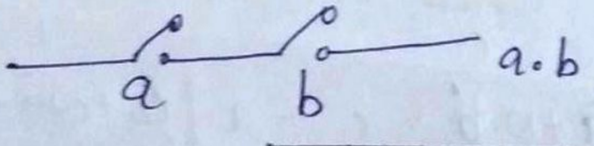
open circuit



closed circuit.

There are two types of Switch formations.

(1) **In Series form:→** two switches a, b are said to in Series form if current flows only when both the switches are in close (on) form.
The output of the switches a and b is denoted by a.b.

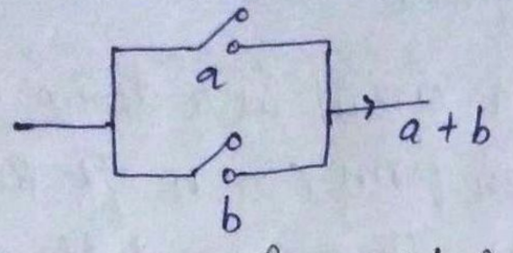


Truth Table :

a	b	a.b
1	1	1
1	0	0
0	1	0
0	0	0

1 → on switch
0 → off switch.

(2) **In Parallel form:→** Two switches a and b are said to be in Parallel form if current flows either one of a or b is in closed form and does not change when both a and b are in open form. The output of the two parallel switches is denoted by a+b.



The Truth Table for $a+b$:

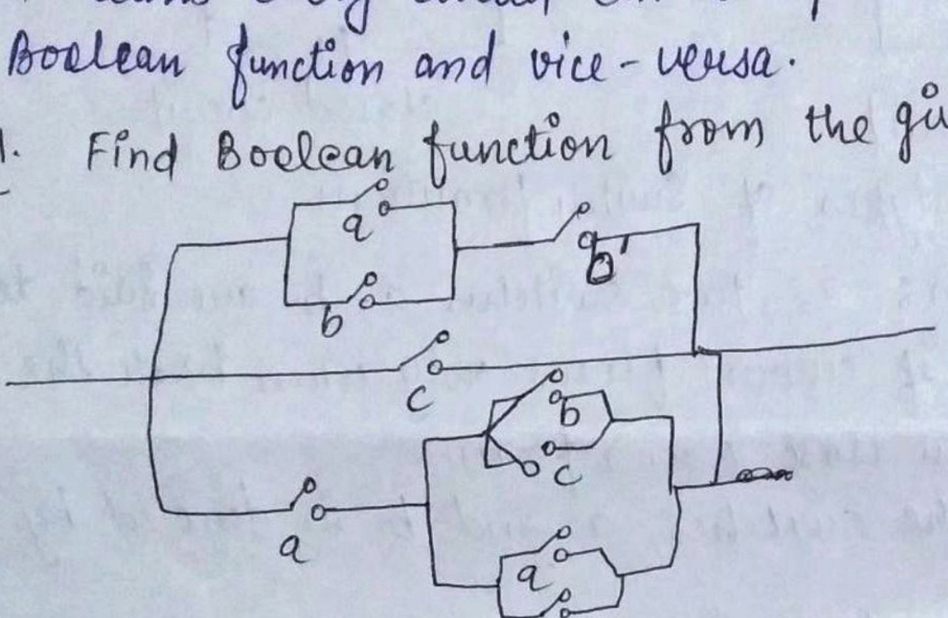
a	b	$a+b$
1	1	1
1	0	1
0	1	1
0	0	0

1 → on switch
0 → off switch

The Algebra of switches is boolean Algebra.

That means every circuit can be represented as a Boolean function and vice-versa.

Ex:-1. Find Boolean function from the given circuit

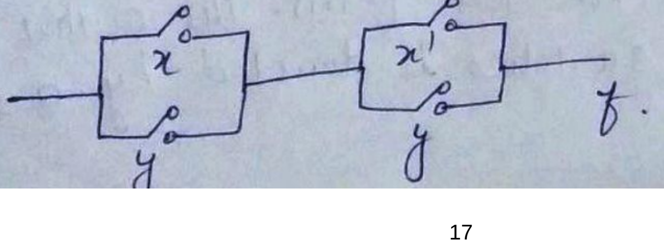


Soln.:- Boolean fn

$$f(a,b,c) = (a+b)\cdot b' + c + a\cdot[(b+c)+(a+c)]$$
$$= (a+b)\cdot b' + c + a\{(b+c)+a\}$$
$$= (a+b)\cdot b' + c + a\cdot(a+b) + a\cdot c$$
$$= a\cdot b' + \underset{0}{b\cdot b'} + c + \underline{a + a\cdot c}$$
$$= a\cdot b' + c + a$$
$$= a\cdot b' + (c+a).$$

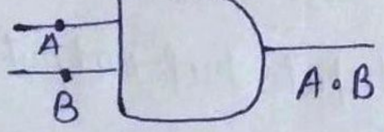
Ex!-2. Draw the circuit of the given Boolean function.

$$f(x,y) = (x+y)\cdot(x'+y)$$



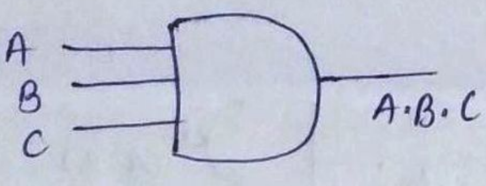
Logic Gates :→ Logic circuits (also called Logic Networks are structures which are built up from certain elementary circuits called logic Gates. In Gates Inputs are two or more than two but output is always one. The elementary logic circuits are as follows:

1. AND Gate :→ In this Gate there are two or more than two inputs but output is always one.
If A and B are two Inputs in AND Gate Then output is Represented by A.B



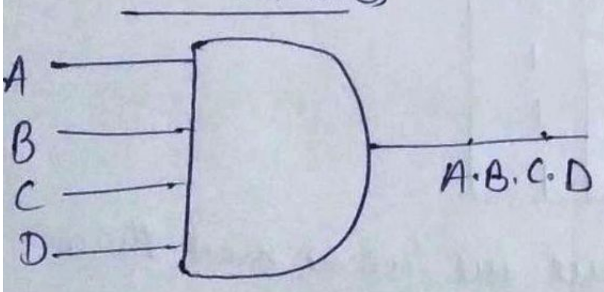
Truth Table

A	B	Y = A.B
1	1	1
1	0	0
0	1	0
0	0	0

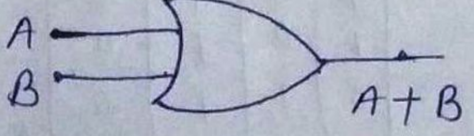


A	B	C	Y = A.B.C
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

Four Variables



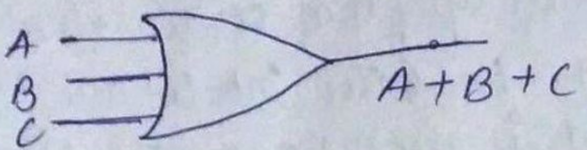
2. OR Gate :— In OR Gate the Inputs may be two or more than two but ~~oput~~ output is always one.
If A & B are two Inputs in OR gate Then output is Represented by A+B.



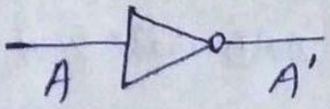
Truth Table

A	B	Y = A+B
1	1	1
1	0	1
0	1	1
0	0	0

For Three variables A, B, C, OR gate is Represented by



(3) NOT Gate :– In this gate, Input is one & output is also one. If A is an Input in this gate Then output is Represented by A'.

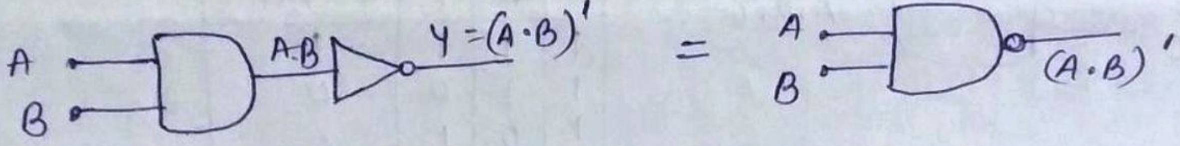


Truth Table

A	A'
1	0
0	1

(4) NAND GAte :–

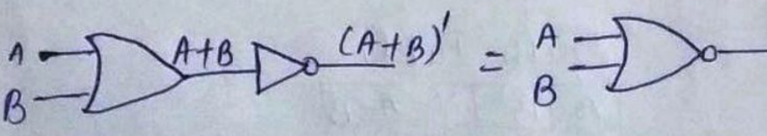
In this gate, there are one or more Inputs but output is always one.
If A & B are two Inputs in this gate, Then output is represented by $(A \cdot B)'$



Truth Table :

A	B	$Y = A \cdot B$	$Y = (A \cdot B)'$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	1

(5) NOR Gate :– In this Gate, There are two or more than two Inputs but output is always one.
if A & B are two Inputs in this Gate. Then output is Represented by $(A+B)'$

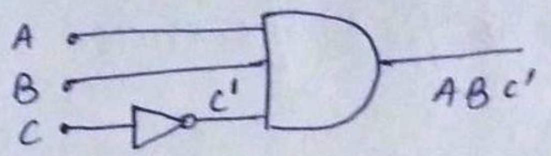


Truth Table :

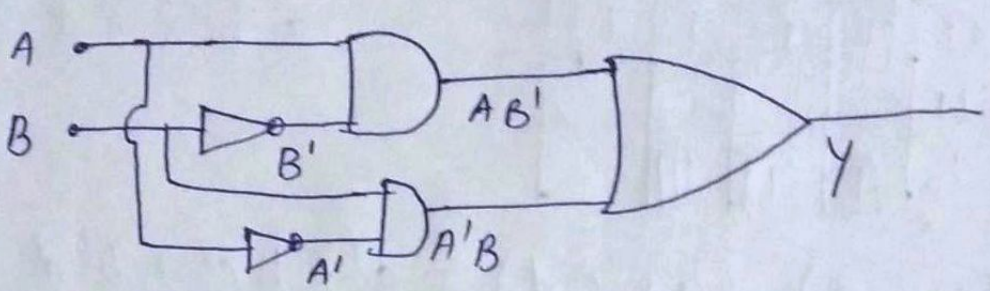
A	B	A+B	$Y = (A+B)'$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1

Ex: Draw the circuit Diagram of Boolean functions

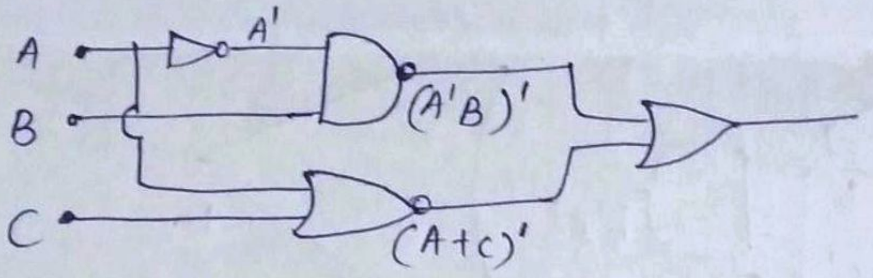
① $F(A, B, C) = ABC'$



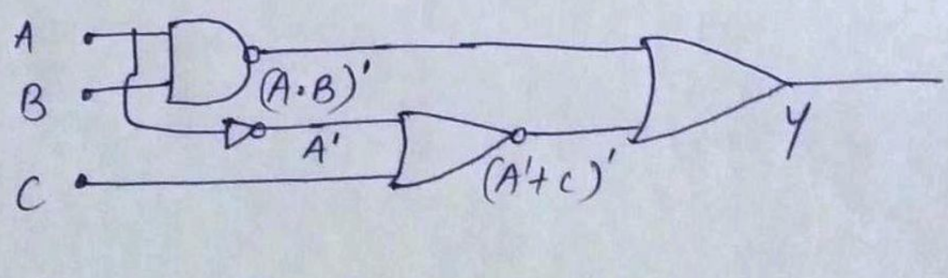
② $F(A, B, C) = Y = AB' + A'B$.



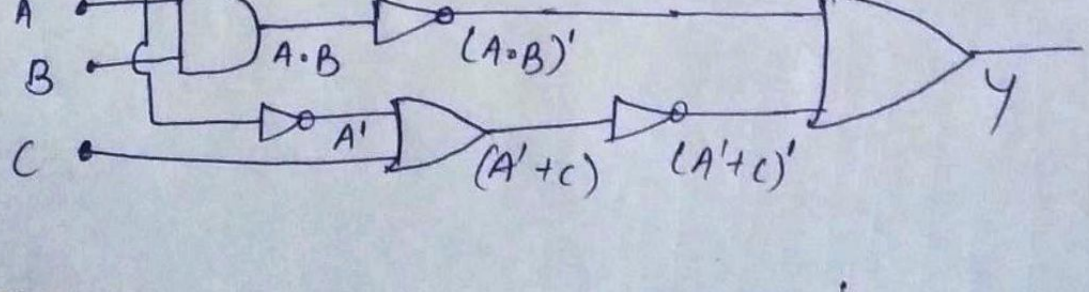
③ $Y = (A'B)' + (A+C)'$



④ $Y = (A \cdot B)' + (A' + C)'$



or



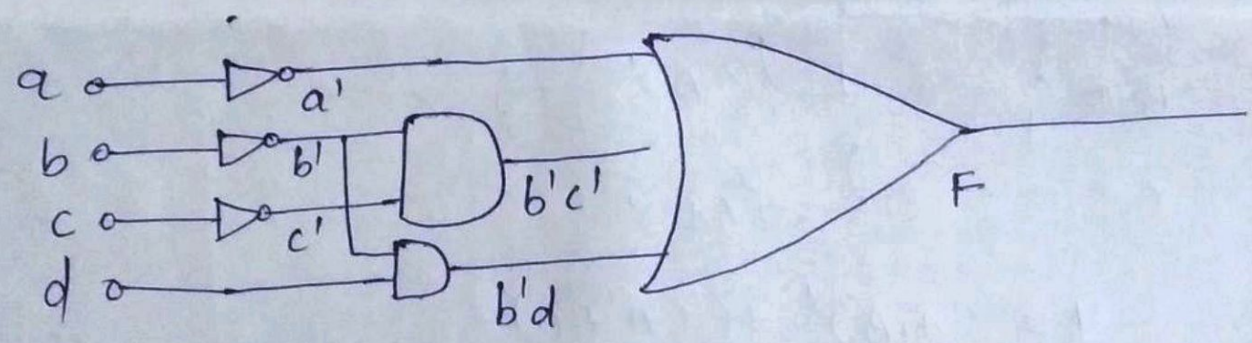
m: 1 Simplify the Boolean function
$f(a,b,c,d) = \Sigma(0,1,2,3,4,5,6,7,8,9,11)$.
using K-Map. Also draw the logic circuits associated with the simplified expression.

ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11				
10	1	1	1	

$a', b', c', d' = 0$

a, b, c & $d = 1$

~~f(x,y,z)~~ $f(a,b,c,d) = a' + b'c' + b' \cdot d$



UNIT - 4

(I.) Propositions, Connectives, Truth Tables: →

(#) **Proposition (Statement):** → A Proposition or Statement is a declarative sentence that is either True or false, but not both.

Examples: 1. lucknow is the capital of state U.P (True)

2. Paris is in France. (True)

3. The Sun Rises in the west. (False)

4. $2 + 6 = 8$ (True)

5. $5 + 6 > 7$ (True)

6. $x + y = 1$ (depends on x & y).

7. open the door (command)

8. what is the colour of Blackboard? (Question)

→ (1), (2), (3), (4) and (5) are statements (Propositions) but (6), (7) and (8) are not statements.

→ Questions, exclamations and commands are not Propositions.

* Propositions are Represented by $p, q, r, \ldots$ and called Propositional variables.

(#) **Compound proposition:** → A Proposition obtained from the combinations of two or more Propositions by means of logical operators or connectives of two or more Prepositions is called compound Proposition.

(#) **Connectives:** → The words and Phrases used to form compound propositions are called connectives.
There are five basic connectives called Negation, conjunction, disjunction, conditional and Biconditional.

(1.) **Negation:** If p is any proposition, the Negation of p, denoted by $\sim p$, $\neg p$ or p' and read as 'not p', is a proposition which is false when p is True and True when p is false. It is uniary operation.

Example: if p : Paris is in France

Then $\sim p$: Paris is not in France

or $\sim p$: It is not the case that Paris is in France.

Truth table for $\sim p$:

p	$\sim p$
T	F
F	T

(2.) **Conjunction:** If p and q are two statements, then conjunction of p and q is the compound statement denoted by $p \wedge q$ and read as "p and q".

$p \wedge q$ is true when both p and q are true, otherwise it is false.

Example: (a) p : The sun rises in the east

q : The sun rises in the west.

Then $p \wedge q$: The sun rises in the east and sets in the west.

(b) p: It is cold & q : It is raining

Then $p \wedge q$: It is cold and raining.

(c) p: Paris is in France

q : $2+6 = 7$

$p \wedge q$: Paris is in France and $2+6=7$.

Truth Table for $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(3.) **Disjunction:** If p and q are two statements, then the disjunction of p and q is the compound statement denoted by $p \vee q$ and read as "p or q".

The statement $p \vee q$ is true if atleast one of p or q is True. It is false when both p and q are false.

Example: If p : It is cold and q : It is Raining then $p \vee q$: It is cold or raining.

Truth Table for $p \vee q$:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table: A Truth table is a table that shows the Truth value (True or false) of a compound proposition for all Possible cases.

Ex: Construct the Truth Table for each compound proposition.

(i) $p \wedge (\sim q \vee q)$ (ii) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$.

(i) Truth table for $p \wedge (\sim q \vee q)$ is given by

p	q	$\sim q$	$\sim q \vee q$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

(ii) The Truth table for $\sim (p \vee q) \vee (\sim p \wedge \sim q)$ is given by.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$(\sim p \wedge \sim q)$	$\sim(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

4. <u>conditional proposition</u>: If p and q are propositions, then the compound proposition 'if p then q' denoted by $p \to q$ and read as 'p implies q' is called a conditional proposition. The proposition p is called antecedent or Hypothesis, and the proposition q is called the consequent or conclusion.

<u>Examples</u>: ① If Today is monday then yesterday was sunday

② if it rains then I will carry an umbrella.

Truth Table for $p \to q$:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(proposition)

5. <u>Biconditional statement</u>: if p and q are two statement (proposition), then the compound statement 'p if and only if q' (p iff q), denoted by $p \leftrightarrow q$ is called a biconditional statement. $p \leftrightarrow q \equiv$ p is necessary and sufficient condition for q.

<u>Examples</u>: ① Ram eats if and only if the food is tasty.

② He swims if and only if the water is warm.

Truth Table for $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \leftrightarrow q$ is True when both of p and q are either true or False.

Converse, Contrapositive and Inverse of a conditional Statement: →

Let p and q are two statements, then the converse, contrapositive and Inverse of $p \to q$ are defined as

(1) <u>Converse</u>: The converse of $p \to q$ is defined as $q \to p$

(2) <u>Contrapositive</u>: The contrapositive of $p \to q$ is defined as $\sim q \to \sim p$

(3) <u>Inverse</u>: The Inverse of $p \to q$ is defined as $\sim p \to \sim q$.

<u>Truth Table</u> for $q \to p$, $\sim q \to \sim p$ & $\sim p \to \sim q$.

p	q	conditional $p \to q$	converse $q \to p$	contrapositive $\sim q \to \sim p$	Inverse $\sim p \to \sim q$	$\sim p$	$\sim q$
T	T	T	T	T	T	F	F
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

<u>Example</u>: Let p and q are two statements as defined by

p: He has the courage

q: He will win

$p \to q$: if He has the courage then He will win.

converse of $p \to q$

$q \to p$: if He wins, then He would have the courage.

contrapositive of $p \to q$

$\sim q \to \sim p$: if He will not win then He has not the courage.

Inverse of $p \to q$

$\sim p \to \sim q$: if He has not courage. Then He will not win.

(#) **Logical Equivalence:→** if two combined Propositions $P(p, q, \cdots)$ and $Q(p, q, \cdots)$, where p & q ... are propositional variables, have the same truth values in every possible case or $P \leftrightarrow Q$ is a tautology, the the propositions are called logically equivalent or simply <u>Equivalent</u>. and denoted by

$$P(p, q, \cdots) \equiv Q(p, q, \cdots)$$

or $P(p, q, \cdots) \Leftrightarrow Q(p, q, \cdots)$

(#) **Algebra of propositions:→**

① <u>Idempotent laws:</u> →
ⓐ $p \vee p \equiv p$
ⓑ $p \wedge p \equiv p$

② <u>Associative laws:</u> →
ⓐ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
ⓑ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

③ <u>commutative laws:</u> –
ⓐ $p \vee q \equiv q \vee p$
ⓑ $p \wedge q \equiv q \wedge p$

④ <u>Distributive laws:</u> –
ⓐ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
ⓑ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

⑤ <u>Identity laws:</u> →
ⓐ $p \vee F \equiv p$ and $p \vee T \equiv T$
ⓑ $p \wedge F \equiv F$ and $p \wedge T \equiv p$

⑥ <u>complement laws:</u> →
ⓐ $p \vee \sim p \equiv T$ and $p \wedge \sim p \equiv F$
ⓑ $\sim T \equiv F$ and $\sim F \equiv T$

⑦ <u>Involution law:</u> → $\sim(\sim p) \equiv p$

⑧ <u>De Morgan's laws:</u> →
ⓐ $\sim(p \vee q) \equiv \sim p \wedge \sim q$
ⓑ $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

⑨ <u>Absorption laws:</u> → ⓐ $p \vee (p \wedge q) \equiv p$ and ⓑ $p \wedge (p \vee q) \equiv q$

Ex:- Show that $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$.
using truth Table and algebra of propositions.

Sol:- (a) <u>using Truth Table</u>

p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	T	F	F	T

Since all the entries in 3rd & last coloums are same therefore $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$

(b) using Propositional Algebra

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\equiv [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$ by distributive law

$\equiv [\sim q \wedge (\sim p \vee q)] \vee [p \wedge (\sim p \vee q)]$ by commutative law

$\equiv [(\sim q \wedge \sim p) \vee (\sim q \wedge q)] \vee [(p \wedge \sim p) \vee (p \wedge q)]$ by distrutive law

$\equiv [(\sim q \wedge \sim p) \vee F] \vee [F \vee (p \wedge q)]$ by complement law

$\equiv \{(\sim q \wedge \sim p) \vee (p \wedge q)$ by Identity law

$\equiv [\sim (p \vee q)] \vee (p \wedge q)$ by de morgan's law

$\equiv (p \vee q) \rightarrow (p \wedge q)$. $[\because p \rightarrow q \equiv \sim p \vee q]$

<u>Hence proved.</u>

Ex:- using truth table show that $p \to q \equiv \sim p \vee q$

and $p \to q \equiv \sim q \to \sim p$

Solⁿ:-

p	q	$\sim p$	$p \to q$	$\sim p \vee q$	$\sim q \to \sim p$	$\sim q$
T	T	F	T	T	T	F
T	F	F	F	F	F	T
F	T	T	T	T	T	F
F	F	T	T	T	T	T

Since the entries of compound proposition $p \to q$ and $\sim p \vee q$ in the truth table are same

therefore $p \to q \equiv \sim p \vee q$.

Also the entries of compound propositions $p \to q$ and $\sim q \to \sim p$ in the truth table are same

therefore $p \to q \equiv \sim q \to \sim p$.

Ex:- Prove the following equivalences by using laws of Propositional algebra.

(a) $(p \to q) \to q \equiv p \vee q$

(b) $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

(c) $(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

Soln:- (a) $(p \to q) \to q \equiv (\sim p \vee q) \to q$

$\equiv \sim(\sim p \vee q) \vee q$ $\quad [\because r \to s \equiv \sim r \vee s]$

$\equiv (p \wedge \sim q) \vee q$ (by demorgan's law)

$\equiv (p \vee q) \wedge (\sim q \vee q)$ (by distributive law)

$\equiv (p \vee q) \wedge T$

$\equiv p \vee q$ (by Identity law)

(b) $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

R.H.S

$(p \to q) \vee (p \to r) \equiv (\sim p \vee q) \vee (\sim p \vee r)$ (by commutive law)

$\equiv q \vee (\sim p \vee (\sim p \vee r))$

(by Associative law)

$\equiv q \vee ((\sim p \vee \sim p) \vee r)$

$\equiv q \vee (\sim p \vee r)$ (by Idempotent law)

$\equiv (\sim p \vee r) \vee q$ (by comm. law)

$\equiv \sim p \vee (r \vee q)$ (by Associative law)

$\equiv \sim p \vee (q \vee r)$ (by comm. law)

$\equiv p \to (q \vee r)$.

(c) L.H.S

$(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv ((\sim p \vee q) \wedge p) \wedge (p \wedge q)$

by Associative law

$\equiv ((\sim p \wedge p) \vee (q \wedge p)) \wedge (p \wedge q)$

by distributive law

$\equiv (F \vee (q \wedge p)) \wedge (p \wedge q)$

by complement law

$\equiv (q \wedge p) \wedge (p \wedge q)$ by Identity law

$\equiv (p \wedge q) \wedge (p \wedge q) \equiv p \wedge q$ by commutative & Idempotent law

Ex:- Show that $[(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))] \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology by using laws of logic.

Sol:- $[(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))] \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$

$\equiv [(p \vee q) \wedge \sim(\sim p \wedge \sim(q \wedge r))] \vee \sim(p \vee q) \vee \sim(p \vee r)$ by using De Morgan's law.

$\equiv [(p \vee q) \wedge (p \vee (q \wedge r))] \vee \sim((p \vee q) \wedge (p \vee r))$ De Morgan's & involution law

$\equiv [(p \vee q) \wedge (p \vee q) \wedge (p \vee r)] \vee \sim((p \vee q) \wedge (p \vee r))$ by using distributive law

$\equiv [((p \vee q) \wedge (p \vee q)) \wedge (p \vee r)] \vee \sim((p \vee q) \wedge (p \vee r))$. by Associative law

$\equiv [(p \vee q) \wedge (p \vee r)] \vee \sim((p \vee q) \wedge (p \vee r))$ by Identity law.

$\equiv P \vee \sim P$ by complement law.

$\equiv T$ where $P = [(p \vee q) \wedge (p \vee r)]$

Ex:- suppose the value of $p \to q$ is false, determine the value (Truth value) of $(\sim p \vee \sim q) \to q$.

p	q	$\sim p$	$\sim q$	$p \to q$	$(\sim p \vee \sim q)$	$(\sim p \vee \sim q) \to q$
T	T	F	F	T	F	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	F	T	T	T	T	F

from truth table, we can see that when the value of $p \to q$ is false, then the value of $(\sim p \vee \sim q) \to q$ is also false.

Tautology:→ A compound proposition that is always true for all possible truth values of its variable, or in other words, that contain only T in the last coloum of its truth Table is called a Tautology.

Contradiction:→ A compound proposition that is always false for all possible truth values of its variables, or, in other words, that contain only F in the last coloum of its truth table is called a Contradiction.

Contingency: A proposition that is neither a tautology nor a contradiction is called a contigency.

i.e a compound proposition that contain T as well as F in the last coloum of its truth table is called a Contingency.

Ex:- Prove that the following propositions are Tautology.

(a) $p \vee \sim p$ (b) $\sim(p \wedge q) \vee q$ (c) $p \rightarrow (p \vee q)$.

Soln:- (a) The Truth Table for $p \vee \sim p$ is given by

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Since the truth value is TRUE for all possible values of the propositional variables which can be seen in the last coloum of the truth table, Hence the given proposition is a tautology.

(b) The Truth table for $\sim(p \wedge q) \vee q$ is given by

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since for any possible assignment of p and q, the Expression $\sim(p \wedge q) \vee q$ is true therefore It is tautology

Ex:- check whether $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is a tautology, contradiction or contingency.

Soln:- The Truth Table for the given compound proposition

r	p	q	$\sim p$	$p \vee q$	$\sim p \vee r$	$(p \vee q) \wedge (\sim p \vee r)$	$(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$	$(q \vee r)$
T	T	T	F	T	T	T	T	T
F	T	T	F	T	F	F	T	T
T	T	F	F	T	T	T	T	T
F	T	F	F	T	F	F	T	F
T	F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

Since all the entries in the coloumn of given compound propositions are TRUE. Therefore The proposition is Tautology.

Ex:- verify that the proposition $p \wedge (q \wedge \sim p)$ is a contradiction.

Soln:- we construct the truth table of the given proposition.

p	q	$\sim p$	$(q \wedge \sim p)$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

for any possible case of p & q, the given proposition is false, which establishes that it is a contradiction.

Well formed formula (WFF):→

- A statement formula is an expression which is a String consisting of variables, Parentheses and connective Symbols.
- A gramtically correct expression is called a well formed formula which can be generated by the following rules:

→ All propositional variables (Individual statement) and constants are wff.

→ If P is wff Then $\neg P$ is also wff.

→ If P and Q are wff, Then $(P \wedge Q)$, $(P \vee Q)$, $(P \to Q)$ and $(P \leftrightarrow Q)$ are all well formed formulae.

→ A Statement formula consists of variables, Parantheses and connectives is recursively wff iff It can be obtained by finitely applying the above Rules.

→ Nothing else is a well-formed formula.

Examples:

wff	Not wff
$\neg(P \vee Q)$	$\neg P \vee Q$
$(P \to (P \wedge Q))$	$(P \wedge Q) \to (\wedge P)$, $\wedge Q$

Ques. construct the Truth Table for the formula
$\sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$	$(p \wedge q)$	$\sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q)$
T	T	F	F	F	F	T	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	T	T	T	F	T

4) Modus Tollens: — The Argument of ...

Tautologically Implication:→

A compound proposition $A(p, q, r, \ldots)$ is said to be tautologically imply or simply imply the compound proposition $B(p, q, r, \ldots)$ if and only if $A \rightarrow B$ is a tautology. This is denoted by $A \Rightarrow B$ and read as 'A implies B'.

Example:— $(p \wedge q) \Rightarrow q$ is tautologically implication. Since $(p \wedge q) \Rightarrow q$ is a tautology. As its truth table contains only T in last column.

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Some Important Tautological Implications: →

1. $p \Rightarrow p \vee q$
2. $\sim p \rightarrow (p \rightarrow q)$
3. $q \Rightarrow (p \rightarrow q)$
4. $\sim (p \rightarrow q) \Rightarrow p$
5. $\sim (p \rightarrow q) \Rightarrow \sim q$
6. $p \wedge (p \rightarrow q) \Rightarrow q$
7. $\sim q \wedge (p \rightarrow q) \Rightarrow \sim p$
8. $\sim p \wedge (p \vee q) \Rightarrow q$
9. $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$
10. $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$.

by Truth Table we can show that these Implications are Tautological Implications.

Argument :→

An Argument is a Process by which a conclusion is drawn from a set of propositions.

<u>Premises or hypothesis</u>: The given set of propositions are called premises.

<u>Conclusion</u>: The final proposition derived from the given Propositions is called conclusion.

Mathematically, An Argument Can be written as

$$\left.\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{array}\right\} \text{Premises}$$

$$\therefore q \leftarrow \text{conclusion.}$$

<u>Valid Argument</u>: An Argument is s.t.b logically Valid argument iff $(p_1 \wedge p_2 \wedge p_3 \cdots \wedge p_n) \rightarrow q$ is a Tautology.

<u>Ex</u>:- An Argument is given as :

If you study hard, Then you get A's grade

If you study hard _ _ _ _ _ _ _ _ _ _

∴ you get A's grade.

Represent the given Argument Symbolically and determine whether the argument is valid.

<u>Soln</u>:- Let p: you study hard

q: you get A's grades

Then symbolically,

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

now have to show $(p \rightarrow q) \wedge p \rightarrow q$ is a tautology

p	q	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since $p \wedge (p \to q) \to q$ is a tautology
Hence the given Argument is Valid.

Rules of Inference:→

The Rules of Inference are criteria for determing the validity of an argument. Any conclusion which is arrived by following these rules is called a valid conclusion, and the argument is called valid argument.

Fundamental Rule 1:– If the statement in p is assumed as true and also the statement $p \to q$ is accepted as true, then q must be true.
Symbolically, It is written in the following Pattern.

$$\left.\begin{array}{c} p \to q \\ p \end{array}\right\} \to \text{Premises or hypothesis.}$$
$$\therefore q \longrightarrow \text{Conclusion}$$

The Rule depicted above is called Modus Ponens or the Rule of detachment.

The validity of the Argument can be seen from the Truth Table: or an argument is valid iff $[(p \to q) \wedge p] \to q$ is a tautology.

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge p \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Ex:— Consider the premises defined as

$p \to q$: if the last digit of this no. is 5, then this no. is divisible by 5.

p : The last digit of this no. is 5.

are True then by Modus Ponens

$$\begin{array}{l} p \to q \\ \underline{p \quad\quad} \\ \therefore q \end{array}$$

q is also true.

Fundamental Rule 2:→ Whenever the two implications $p \to q$ and $q \to r$ are accepted as True, then the implication $p \to r$ is also accepted as True.

Symbolically, It can be represented as

$$\left.\begin{array}{l} p \to q \\ q \to r \end{array}\right\} \to \text{Premises or hypothesis}$$

$$\therefore p \to r \longrightarrow \text{Conclusion.}$$

This Argument is known as Hypothetical Syllogism.

The truth table for this Argument is as follows.

$p \to r$	p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T
T	T	F	T	F	T	F	T
F	T	F	F	F	T	F	T
T	F	T	T	T	T	T	T
T	F	T	F	T	F	F	T
T	F	F	T	T	T	T	T
T	F	F	F	T	T	T	T

Since $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

Hence this Argument is valid.

Ex:- Represent the Argument.

If it rains today, then we will not have a Party today

If we do not have Party today, then we will have a Party tomorrow.

- -

Therefore, if it Rains today, then will have a Party tomorrow.

Symbolically and determine whether the argument is valid

Sol^n:- Let

p: It rains today

q: We will not have a Party today

r: We will ~~no~~ have a party tomorrow

Then the given argument can be Represent as

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore\ p \rightarrow r \end{array}$$

Hence, by Hypothetical Syllogism

The Argument is valid.

Modus Tollens: — The Argument of the form

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

This Argument is valid and is called modus tollens.

Ex:- Represent the argument

If this no. is divisible by 6, then it is divisible by 3.
This no. is not divisible by 3

This no. is not divisible by 6.

Symbolically, and determine whether the Argument is valid.

Soln:- If we let

p: The no. is divisible by 6

q: It is divisible by 3.

The argument may be written as

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

Thus by modus tollens the argument is valid.

Disjunctive Syllogism: —

The following statement form is valid

$$\begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

This argument states that when there are two possibilities and one can rule one out, the other must be case.

Ex:- Represent the argument,

Either Ram is not guilty or Shyam is telling the truth
Shyam is not telling the truth

Ram is not guilty.

Sol^n:- If we let

p: Ram is not guilty, q: Shyam is telling the truth.

Then the argument can be written as

$$\begin{array}{c} p \vee q \\ \sim q \\ \hline \therefore\ p \end{array}$$

Thus, by disjunctive syllogism, the argument is valid.

Example: Prove the validity of the following argument. If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, I will not get the job or I will not work hard.

Solⁿ:- Let

p: I get the job
q: I work hard
r: I get promoted
s: I will be happy.

Then the given arguments can be written in symbolic form as

$$\left.\begin{array}{c}(p \wedge q) \rightarrow r \\ r \rightarrow s \\ \sim s \end{array}\right\} \rightarrow \text{Premise (given)}$$

$$\therefore \sim p \vee \sim q.$$

Now applying some Rule of Inferences.

1. $(p \wedge q) \rightarrow r$ Premise (Given)
2. $r \rightarrow s$ Premise (given)
3. $(p \wedge q) \rightarrow s$ Hypothetical syllogism using 1 & 2.
4. $\sim s$ Premise (given)
5. $\sim (p \wedge q)$ Modus Tollens using 3 & 4
6. $\sim p \vee \sim q$ conclusion de Morgan's law.

Hence the argument is valid.

Rule of Inference	Tautological form	Name
1. $\dfrac{p}{\therefore p \vee q}$ & $\dfrac{q}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$ & $q \rightarrow (p \vee q)$	Addition
2. $\dfrac{p \wedge q}{\therefore p}$ & $\dfrac{p \wedge q}{\therefore q}$	$(p \wedge q) \rightarrow p$ & $(p \wedge q) \rightarrow q$	Simplification
3. $\dfrac{p \quad q}{\therefore p \wedge q}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
4. $\dfrac{p \rightarrow q \quad p}{\therefore q}$	$[(p \rightarrow q) \wedge p] \rightarrow q$	Modus Ponens.
5. $\dfrac{p \rightarrow q \quad \sim q}{\therefore \sim p}$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$	Modus tollens.
6. $\dfrac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical Syllogism.
7. $\dfrac{p \vee q \quad \sim p}{\therefore q}$	$[(p \vee q) \wedge \sim p] \rightarrow q$	Disjunctive Syllogism.
8. $\dfrac{p \rightarrow q \wedge (r \rightarrow s) \quad p \vee r}{\therefore q \vee s}$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) \rightarrow (q \vee s)$	constructive dilemma
9. $\dfrac{p \rightarrow q \wedge (r \rightarrow s) \quad \sim q \vee \sim s}{\therefore \sim p \vee \sim r}$	$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s) \rightarrow (\sim p \vee \sim r)$	Destructive dilemma.

consistent premises :→

A set of premises $P_1, P_2, \ldots, P_n$ are said to be consistent if their conjunction $P_1 \wedge P_2 \wedge \ldots P_n$ has the truth value 'T' in atleast one possible situation.

Inconsistency of Premises :→

A set of premises $P_1, P_2, \ldots, P_n$ are said to be Inconsistent if their conjunction $P_1 \wedge P_2 \wedge \ldots \wedge P_n$ has the Truth value 'F' in every possible situation.

Example: ① Show that the premises $p \to q$, $p \to r$, $q \to \sim r$, p are inconsistent.

Solution:→ The Truth Table for the given set of premises is as follows:

p	q	r	$\sim r$	$p \to q$	$p \to r$	$q \to \sim r$	$(p \to q) \wedge (p \to r) \wedge (q \to \sim r) \wedge p$
T	T	T	F	T	T	F	F
T	T	F	T	T	F	T	F
T	F	T	F	F	T	T	F
T	F	F	T	F	F	T	F
F	T	T	F	T	T	F	F
F	T	F	T	T	T	T	F
F	F	T	F	T	T	T	F
F	F	F	T	T	T	T	F

Since the conjunction of given premises has the Truth value 'F' in all possible situation Therefore the premises $p \to q$, $p \to r$, $q \to \sim r$, p are inconsistent.